A Cost-Efficient Scheduling Algorithm for Traffic Grooming

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Abstract. In this paper, we consider a fundamental traffic grooming problem in optical line-system: the number of total links with lengths and the max number of wavelengths (capacity) of each fiber are given, also a set of demands (jobs) and their routes are fixed so that the load of each link is known, the problem is to construct a set of fiber intervals so that the total fiber length is minimized (called Fiber Lengths Minimization problem or FLM for abbreviation). It is known that FLM problem is NP-complete in general. In this paper, we propose a 2-approximation algorithm, Longest Link interval First (LLF), which is better than existing best known bound.

Keywords: Traffic Grooming, Fiber Lengths Minimization problem, Minimizing Total Fiber Length, Longest Link interval First (LLF).

1 Introduction

In optical network design, decomposing the network into a set of optical line systems is one way of avoiding the expensive O-E-O (optical to electrical to optical) conversion [13]. In this way, system becomes transparent, only demands between different linesystems need O-E-O conversion; also routing is not necessary in this case since wavelength assignment problem can be solved separately in each linesystem. All-optical networks have been extensively studied in recent years, especially for the core networks. A logical path formed by a signal traveling from its source to its destination using a unique wavelength is termed a lightpath. If the nodes have no conversion capability, then the requirement that the same wavelength must be used on all the links along the selected route is known as the wavelength continuity constraint and makes networking significantly different from conventional circuit switched networks. Our work assumes that the nodes are not capable of wavelength conversion.

The network usually supports traffic that is at rates that are lower than the full wavelength capacity, and therefore the network operator has to be able to put together (groom) low-capacity connections into the high capacity lightpaths. The network operator often has to groom low-capacity demands into high capacity

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fibers to save cost (energy and equipment costs etc.). This can be viewed as assigning colors to the lightpaths so that at most g of them can share the same fiber, where $g \geq 1$ is the capacity of a single fiber. This grooming problem is one of very important issues in optimizing costs for optical networks. With MOADMs (mesh optical add/drop multiplexer), there is polynomial time solution for this problem [13].

Without MOADMs, the linesystem can be treated as a collection of fibers each of which occupies an interval of the line, the problem is called packing intervals in intervals, which is NP-complete (see a proof given in [13]; in this case, each demand must be assigned not only a wavelength but also a fiber which covers the intended interval; deploying fiber satisfying all demands but minimizing total length (MinLength) of fibers is NP-hard when total number of wavelength (μ) is larger than 1. In this paper, the case without MOADMs and wavelength conversion is considered. The book [3] provides many research results about scheduling algorithms that may be applied in job allocations. The paper [13] discusses wavelength assignment and generalized interval graph coloring and provides NP-complete proof for the problem. [8] reviews recent research into the energy-efficiency in optical networks. [7] summarizes recent technologies for reducing the power consumption of optical access networks. [9] [5] discusses the regenerator placement and routing in translucent optical networks. [4] provides approximating solution for traffic grooming with respect to ADMs and OADMs. [2] provides a $(logM)$ and $(log\mu)$ -approximation algorithms for minimizing total number of fibers where M is the number of links in this system. [6] proposes a general 4-approximation algorithm for minimizing total number of OADMs. [10] discusses the online version of this scheduling problem. In this paper, 2-approximation algorithm is proposed.

2 Problem Formulation

The problem can be formally stated as follows: an optical line-system has n links $e_1, e_2, \ldots, e_n$, with link e_i carrying fibers and each fiber can carry g wavelengths, the length of link e_i is L_i. Represent a demand by $[i, j]$ for $i \leq j$ if it requires links $e_i, \ldots, e_j$; the set of demands (jobs) will be denoted by D. The load $l_i = l(e_i)$ on link e_i is the minimum number of fibers required to carry all the demands on link e_i, where d_i is the number of demands on link e_i. Consider demands D are given, together with link lengths, the objective is to construct a set F of fiber intervals of minimum total length which can satisfy D, it is called Fiber Lengths Minimization problem (FLM problem for abbreviation).

Theorem 1. *The lower bound for FLM problem is the sum of the minimum number of fibers used on each link multiplies the length of each link.*

Proof: For a given set of jobs J and demands D, we can find the minimum number of fibers needed for each link, denoted as $l_i, l_2, \ldots, l_k$,

$$l_i = l(e_i) = \lceil \frac{d_i}{g} \rceil \tag{1}$$

for total k links under consideration, where l_i is the minimum number of fibers needed for link e_i. Then ideally, the min length of all fibers is the sum of (the minimum number of fibers used in each link) multiplies (the length of each link (denoted as L_i)), i. e. :

$$MinLength(OPT) = \sum_{i}^{n} l_i L_i \tag{2}$$

Example 1: As shown in Fig.1. , there are four requests j_1 to j_4 and three

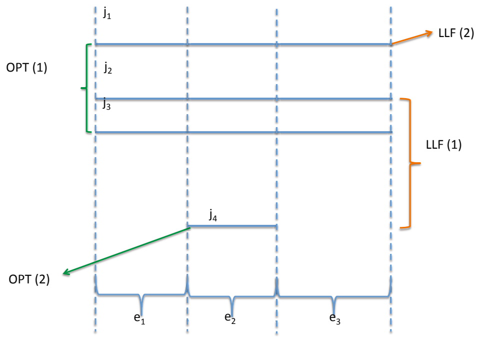

Fig. 1. An example of MinLength problem

links e_1, e_2, e_3. j_1, j_2 and j_3 pass through link e_1, e_2 and e_3; j_4 passes through link e_2. Each fiber has wavelengths (capacity) $g = 3$. Therefore, the lower bound of total number of fibers needed on link e_1 to e_3 is 1, 2, 1 respectively; and the total length of all fibers is ($L_1 + 2L_2 + L_3 = l_1L_1 + l_2L_2 + l_3L_3$), i. e., the sum of (the minimum number of fibers used in each link) multiplies (the length of each link (denoted as L_i)).

Observation 1: The lower bound for FLM problem is to allocate exactly $\lceil \frac{d}{g} \rceil$ number of fibers to each link, where d_i is the fiber length on link i.

Remark 1: The lower bound is not easy to achieve. One way to achieve this is to apply First Fit Decreasing (FFD) algorithm in [6] to sort all requests in non-increasing order of their spans, and allocate the subset of longest span jobs first. By sorting all requests in non-increasing order of their spans and allocate the subset of longest span jobs first, the long span jobs will not be distributed to too many other fibers, so that the total fiber length may be minimized. However, FFD may not work well in some cases. It is shown that FFD has approximation ratio 4 in the worst case [6].

In the following, we consider that the nodes have no wavelength conversion capability. The paper [6] showed that it is NP-hard to approximate our problem already in special case where all jobs have the same span and can be allocated on one fixed link, by a simple reduction from the subset sum problem. To see the hardness of the FLM problem, THEOREM 2 is given as follows:

Theorem 2. *FLM problem is NP-complete problem in general case.*

We sketch a simpler proof than [10] as follows:
Proof: In the following, we show that the well-known NP-complete problem, K-PARTITION problem can be reducing to FLM problem in polynomial time. K-PARTITION problem is well-known NP-complete (see [3] and reference therein): for a given arrangement S of positive numbers and an integer K. Partition S into K ranges so as to sums of all the ranges are close to each other. K-PARTITION problem can be reduced to our FLM problem as follows. For a set of jobs J, each has capacity demand and span constraints (set as positive numbers), partitioning J by their capacities into K ranges, is the same to allocate K ranges of jobs with capacity constraint g (i.e. the sum of each range is at most g). On the other hand, if there is a solution to K-PARTITION for a given set of intervals, there exists a schedule for the given set of numbers. Since K-PARTITION is NP-hard in the strong sense, our problem is also NP-hard. In this way, we have found that that our FLM problem is NP-complete problem.

Definition 1. *Approximation ratio: an offline deterministic algorithm is said to be C-approximation for the objective if it obtains results in a polynomial time at most C times that of an optimal solution.*

Since the general FLM problem is NP-complete, in the following, we propose an efficient approximation algorithm.

3 The Approximation Algorithm: Longest Link Interval First

In this section, a 2-approximation algorithm called Longest Link interval First (LLF) is introduced. The LLF algorithm is described in Algorithm 3.1. The LLF algorithm allocates the requests from the longest link interval to the shortest interval. Each job is scheduled to the first fiber which can fit. This algorithm has computational complexity $O(N \max(M, logN))$ where N is the number of jobs and M is the number of fibers needed on any link. Because The LLF algorithm firstly sorts all jobs (requests) in non-decreasing order of their start points (line 1), this takes $O(NlogN)$ time. The load of each link is represented by min number of fibers needed (line 2-4). Then the algorithm finds a fiber for a request needs $O(M)$ steps where M is the min number of fibers need on any link (line 5-12), N jobs altogether need $O(MN)$ steps. Therefore, the entire algorithm takes $O(N\ max(M, logN))$ time where normally $N > M$.

Example 2: As shown in Fig. 1 where each fiber can carry max g=3 wavelengths. Without loss of generality, assuming that link length $L_2 > L_1 = L_3$. According

Input: A job (demand) instance $J = \{j_1, j_2, \ldots, j_n\}$, and the max capacity g of a fiber (g is the grooming papameter)
Output: The allocated jobs and total length of all fibers
Sort all jobs in non-decreasing order of their start-points (s_i for job i), such that $s_1 \leq s_2 ... \leq s_n$, set f=1 **forall the** links under consideration **do**
  represent load of link e_i by the min number of fibers needed, denoted as l_i (take integral value by ceiling function).
end
forall the jobs under consideration **do**
  Find the longest continuous link interval with same load first, denoted as $[z_1, z_2]$; If two link interval have same length, consider larger load first
  forall the jobs ended or started in $[z_1, z_2]$ **do**
    always consider the longest job when other parameters are the same;
    allocate to the first fiber which can fit, use a new fiber and set f=f+1 if needed
  end
  remove allocated jobs, update load of each link
end
Count load of all links and total length of all fibers.

Algorithm 3.1: Longest Load First Algorithm

to LLF algorithm, j_4 is allocated firstly to the first fiber on link e_2 since the longest link interval is on it, j_2 and j_3 are also allocated to the first fiber on link e_2 since j_1, j_2 and j_3 have the same start-point and length. j_1 is then allocated to another fiber on link e_2 since g=3 on any fiber. Notice that j_1, j_2 and j_3 can be allocated in any order in this case.

Theorem 3. *The approximation ratio of our proposed Longest link interval First(LLF) algorithm for FLM problem has an upper bound 2.*

Proof: We provide a proof by induction. Consider there are n requests and a fiber can carry g wavelengths.

1. Since one fiber can host at most g requests, let firstly consider n=$g+1$, we have $LLF(J) \leq 2OPT$ in this case. The adversary is that these $g+1$ jobs have different start-points, end-points, shorter jobs are contained by the longer ones, and are sorted in non-decreasing order of their start-points as shown in Fig. 3(b) where f is set as 2 in this case. The total fiber length of optimal solution is dominated by the length of the longest job with span T_1, (g+1)-th job with span T_{g+1} (assuming it is the shortest span length but has link length longer that other links). LLF treats most-load links first when two load spans have same length, its total fiber length is dominated by the 2-nd longest job with span T_2, and the longest job with span T_1 (one job left for a single fiber). Therefore we have:

$$\frac{LLF(J)}{OPT(J)} = \frac{T_1 + T_2}{T_1 + T_{g+1}} = \frac{1 + \frac{T_2}{T_1}}{1 + \frac{T_{g+1}}{T_1}} \tag{3}$$

Equ. (3) will have upper bound 2 when T_1=T_2 and other span lengths are negligible comparing to T_1; for other cases, LLF(J) equals to OPT(J).

2. Assuming that $LLF(J) \leq 2\ OPT(J)$ holds for n=k under clique, one-sided clique, container and other cases. And there are total f fibers used. Let us denote the optimal solution and LLF solution as OPT_k and LLF_k respectively, we have:

$$LLF_k \leq 2OPT_k \tag{4}$$

3. Next, we consider n=k+1. For this case, there are following situations after sorting all $k+1$ jobs in increasing order of their start-points:

   (a) The total number of fibers needed is still f, i.e., the $(k+1)$-th job can be allocated to one of f existing fibers. There are following two sub conditions:
   i). The $(k+1)$-th job can be allocated to one of f existing fibers and the total fiber length of all fibers will not change, i.e., LLF_k=LLF_{k+1} and $OPT_k = OPT_{k+1}$. In this case, obviously, $LLF_{k+1} \leq 2OPT_{k+1}$ holds.
   ii). Assuming that the allocation of $(k+1)$-th job will increase the total fiber length of LLF and OPT by l_{k+1} for the upper bound. i. e., LLF_{k+1}=$LLF_k + l_{k+1}$, OPT_{k+1}=OPT_k+l_{k+1}, and $l_{k+1} \leq len(j_{k+1})$. (As for other scenarios, such as the $(k+1)$-th job only increases the total fiber length of LLF (i.e., $(k+1)$-th job is contained by some longer jobs) or only increases the total fiber length of OPT, one can easily check that $LLF_{k+1} \leq 2OPT_{k+1}$ holds). We then have:

$$LLF_k + t_{k+1} \leq 2OPT_k + t_{k+1} \leq 2OPT_k + 2t_{k+1} \leq 2OPT_{k+1} \tag{5}$$

   (b) The total number of fibers needed will increase by 1, i.e., $(f+1)$ fibers are needed. This means that the $(k$+$1)$-th job intersects with all existing jobs and cannot be hosted by any existing fiber. We consider the following three typical hard sub-conditions (other scenarios are trivial and easy to show so that we omitted the proofs for them) :
   i). One-sided clique: in this case, all job intervals form a one-sided clique, either started or ended at the same node as shown in Fig.2, assuming link e_1 has longest length. In this case, optimal solution is to allocate longest group of jobs to a fiber, the second longest group of jobs to another fiber, and so on. The total fiber length of optimal solution is dominated by the span length of the longest job with span T_1, $(fg+1)$-th job with span T_{fg+1} (the shortest one). Let us denote total length of other fibers as T_O in optimal solution. LLF treats most-load links first when two load span have same lengths, its total fiber length is dominated by the 2-nd longest job with span T_2, and the longest job with span T_1 (one job left for a single fiber), denote total length of other fibers as T_H. therefore:

$$\frac{LLF_{k+1}}{OPT_{k+1}} = \frac{T_1 + T_2 + T_H}{T_1 + T_{g+1} + T_O} = \frac{1 + \frac{T_2+T_H}{T_1}}{1 + \frac{T_{g+1}+T_O}{T_1}} \tag{6}$$

Equ. (6) will have upper bound 2 when $T_1{=}T_2$ and other span lengths are negligible comparing to T_1; for other cases, LLF_{k+1} equals to OPT_{k+1}.

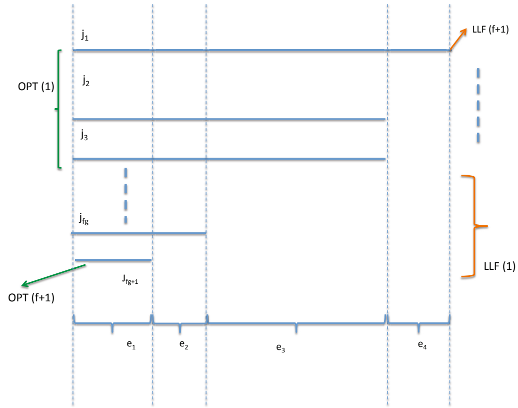

Fig. 2. One-sided clique case for LLF algorithm

ii). Clique case: Let us consider all jobs started and ended at different time as shown in Fig. 3 (a). The adversary is that two or more jobs with longest spans are spreading left and right across the point where all job intervals intersect, assuming the center link e_x (the cross point for all jobs) has longest length. Optimal solution will allocate others to one or more fiber and allocate j_{g+1} to a separate fiber with total fiber length of $(T_{g+1}{+}T_g{+}T_O)$; LLF algorithm will allocate j_{g+1} and j_g to one fiber, so on and the shortest one left for a single fiber, let set it as j_1. therefore:

$$\frac{LLF_{k+1}}{OPT_{k+1}} = \frac{T_{g+1} + T_g + T_H + T_1}{T_g + T_{g+1} + T_O} = \frac{1 + \frac{T_H + T_1}{T_g + T_{g+1}}}{1 + \frac{T_O}{T_g + T_{g+1}}} \tag{7}$$

Equ. (7) will have upper bound 2 when $T_1{+}T_H{=}T_g{+}T_{g+1}$ and other span lengths are negligible comparing to $T_g + T_{g+1}$; for other cases, LLF_{k+1} equals to OPT_{k+1}. A similar 2-approximation algorithm by consider span distance is also provided in [6] for this case.

iii). The container case: The adversary is shown in Fig. 3 (b), i.e., shorter interval jobs are contained in longer interval jobs and assuming link e_x is longer than other links, this is one of the worst cases for LLF algorithm. Let us set these $(k+1)$ jobs have lengths T_1, T_2, ..., T_k, T_{k+1} in non-increasing order. The $(k+1)$-th job is the longest jobs for LLF, so

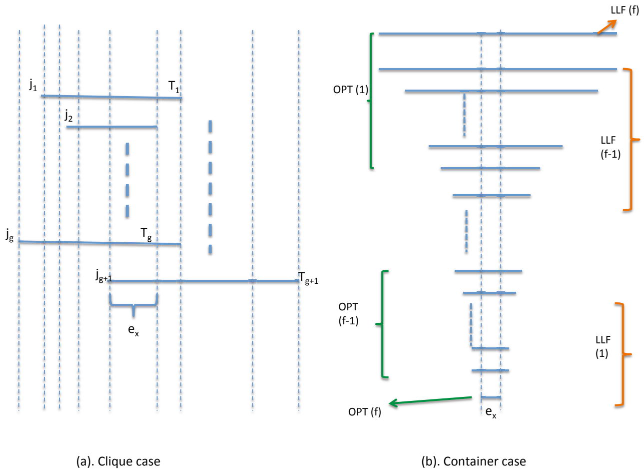

Fig. 3. Clique and Container Case for LLF algorithm

that $LLF_{k+1}=(T_{k+1}+LLF_{k-1})+T_1=T_{k+1}+LLF_k$. As for the optimal solution, one can allocates the longest job first, so $OPT_{k+1}=OPT_k+T_{k+1}$. Therefore $LLF_{k+1}=LLF_k+T_{k+1} \leq 2OPT_k+T_{k+1} \leq 2OPT_k+2T_{k+1}=2OPT_{k+1}$, this means $LLF_{k+1} \leq 2OPT_{k+1}$.

By combining the above analyses, we have proved Theorem 3.

4 Conclusion

In this paper, an efficient traffic-grooming algorithm, LLF, for minimizing total fiber length is proposed. Both theoretical lower bound and approximation are discussed. The proposed algorithm can help network designer to save the deployment cost, management cost and energy etc. We are still looking for near-optimal solution for this problem. There are a few more open research issues for the problem: including finding better near-optimal solution and providing theoretical proofs for the approximation algorithms; extending to other network topologies, like in [1][2][6]; considering stochastic demands such as in [11] [12]; and considering other optimization objectives. With the above-mentioned extensions and other related issues, it is possible to develop comprehensive cost-efficient methods for traffic grooming in optical networks.

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